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Jacobsen, H. Moust

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DILATANCY AND COHESION IN FRICTIONAL MATERIALS

H. Moust Jacobsen, University of Aalborg, Denmark.

SYNOPSIS: When estimating the strength parameters of a sand mineral by sliding two blocks against each other, the sand mineral appears to behave as a pure frictional material. However, when sand specimens are tested using triaxial equipment, an effective cohesion c' is measured due to the effect of dilation during failure. It is this close connection between dilation and effective cohesion which will be demonstrated using a simple model of regularly packed cylinders.

Introduction

Analysis of the ultimate equilibrium of soil masses, carrying loads from buildings or other structures, has been one of the main problems in geotechnical engineering for many years. The analysis is normally based on laboratory testing with the ultimate resistance of the soil specimen described by a failure criterion such as Coulomb-Mohr's or von Mises'. In the last two decades new apparatus has been developed, and electronics has been introduced in laboratory and field equipment. This has considerably increased the amount of test data. This large quantity of test data is leading to a better understanding of the complex behaviour of soil and to even more complicated failure criterions. However, this behaviour must be described as simply as possible in order to facilitate theoretical or practical analysis.

Today's theoretical methods are normally based on theories which have been developed during the last two centuries, beginning with the fundamental considerations of Coulomb in 1776. All major problems can be analysed theoretically in the plane state if the boundaries are uncomplicated. Analysis requires the soil to be described as having very simple properties. For instance the theory of elasticity uses only one deformation modulus, and the theory of plasticity normally uses a single value of a strength parameter. Even numerical methods, such as the finite element method, are normally based on very few material parameters. Nevertheless, these methods can be used to predict displacements of soil grains even in very complex stress and strain states.

Many problems are solved by performing tests, either small scale model tests or full scale tests. The development of electronic equipment has made it possible to measure porepressure, stresses and displacements very accurately. By requiring similarity the results can be directly associated with the actual condition without further analysis. But the soil's behaviour depends very much on the stress level which can have an impact on the application of similarity.

Therefore, it is very useful to achieve a detailed knowledge of soil behaviour, but it is also important to use this knowledge to find a simple and appropriate set of parameters which can be handled in theoretical analysis.

Soil behaviour during failure

The strength parameters are normally measured in a triaxial compression test or in a plane shearbox test. But in the last twenty years special test equipment has been developed (for instance by Green (1971), Lade (1972) or Bønding (1973)). In this equipment the three principal stresses and strains, acting on a cubical specimen, can be independently controlled. The intermediate principal stress can then be varied between the minor and major principal stresses.

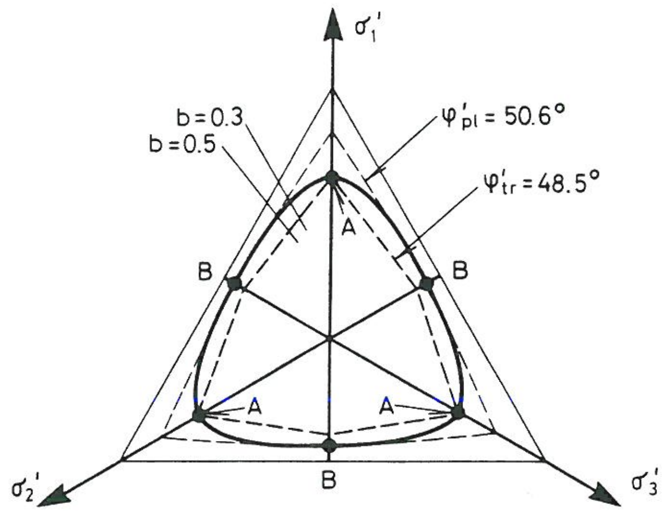

Fig. 1. Results of cubical triaxial tests. A: Triaxial compression. B: Triaxial extension.

Test results can be shown in three-dimensional stress space by using the octahedral plane as shown in Fig. 1. It is based on Lade (1984) with the exception that the points which mark the test results have been cut out.

Several numerical methods employ the Extended Tresca criterion to describe failure. When drawing in the octahedral plane the Tresca criterion forms a regular hexagon as shown by the test results in Fig. 1. This makes for a rather poor description. The Extended von Mises (or Drucker-Prager) forms a circle with its center on the hydrostatic axes; this obviously does not describe the test results any better than the Extended Tresca does.

Paul Lade (1977) has proposed a failure criterion based on the first and third stress invariants of the stress tensor I_1 og I_3:

$$(I_1^3/I_3 - 27)\,(I_1/p_a)^m = \eta_1$$

where p_a is atmospheric pressure expressed in the same units as I_1. The strength parameters are m and η_1. The curved line in Fig. 1 corresponds to $\eta_1 = 104$ and $m = 0.16$. This is in perfect agreement with the test results.

The Coulomb failure criterion forms an irregular hexagon. Two of these hexagons are shown in Fig. 1 corresponding to $\varphi' = 48.5°$

and $\varphi' = 50.6°$. If triaxial compression tests are used to estimate φ' ($\varphi' = 48.5°$), the Coulomb failure criterion gives the dotted line inside the curve. Thus, theoretical analysis based on triaxial tests would always be on the safe side. If a higher value of $\varphi'(\varphi'_{pl} = 50.6°)$ is used, then Coulomb's failure criterion would cover nearly the entire test result, with the exception of the stress states which are close to the triaxial compression state.

The stress state can be characterized by the parameter b, Green (1972):

$$b = \frac{(\sigma'_2 - \sigma'_3)}{(\sigma'_1 - \sigma'_3)}$$

where $b = 0$ corresponds to triaxial compression (A-points in Fig. 1) and $b = 1$ to triaxial extension (B-points in Fig. 1). When in the plane state, b is found to range from 0.2 to 0.5 with a mean value of 0.3. Fig. 1 then shows that when a certain value of φ is used, which is higher than that measured during triaxial compression tests, Coulomb's failure criterion covers the stress states varying from the plane state $b = 0.3$ to the triaxial extension state $b = 1$. Following this line of logic, a very simple method can be used:

i) Estimate the angle of friction φ'_{tr} using normal triaxial tests.

ii) Then increase it to a higher value φ'_{pl} which corresponds to the plane state.

Since most theoretical analysis takes place in the plane state the divergences for values of $b \leq 0.2$ become insignificant. Fig. 1 shows that this procedure agrees well with even the most accurate investigations of the three dimensional failure state.

The difference between the triaxial state and plane state has been studied by Green (1972) and Bønding (1973), among others. Green has shown that the friction angle depends on the stress state parameter b and the porosity n (or density index I_D). The plane state corresponds to $b \sim 0.2$ for a dense Ham River sand and to $b \sim 0.4$ for a loose Ham River sand. The results from Green's paper can be expressed by:

$$\varphi'_{pl} = \varphi'_{tr}(1 + k_\varphi \cdot I_D) \qquad (1)$$

and Fig. 2 shows that $k_\varphi = 0.13$ for Ham River sand. Bønding estimated $k_\varphi = 0.16$ for G-12 sand.

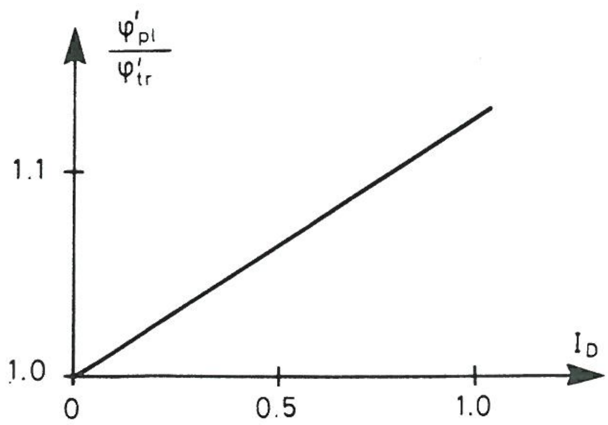

Fig. 2. Idealized relationship between friction angles measured in plane and triaxial compression states.

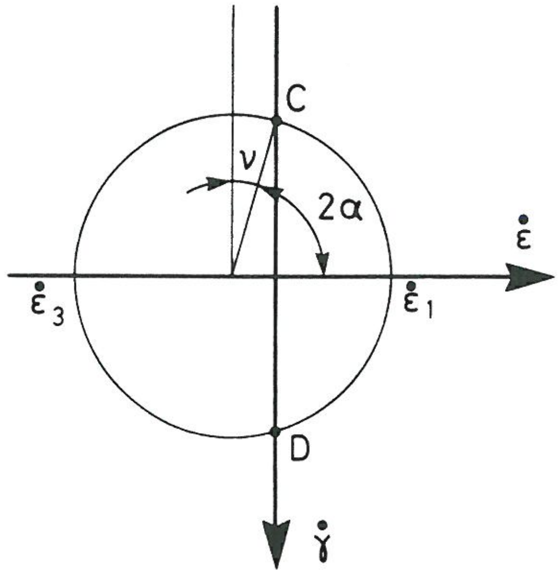

Fig. 3. Mohr's circle of strain increment. Definition of angle of dilation.

This correction has been used successfully in Denmark for many years. Model tests on sand (Hansen and Odgaard, 1960) have shown that the ultimate bearing capacity of a shallow strip footing corresponds to a value somewhat higher than that of the angle of friction when measured in triaxial tests. Therefore, the use of $\varphi'_{pl} = 1.1\varphi'_{tr}$ was recommended by J. Brinch Hansen in the Danish Code of Practice for Foundation Engineering (1965).

The difference between φ'_{pl} and φ'_{tr} seems to be rather small, but its importance can be easily demonstrated. For instance, the ultimate bearing capacity of a foundation will normally be more than double when using φ'_{pl} instead of φ'_{tr}.

During failure the plastic strains totally dominate the elastic strain. Fig. 3 shows the Mohr's circle of plastic strain increments for a soil which is expanding during failure. The angle of dilation ν is a characteristic parameter of soil behaviour just as φ' and c'. (Bent Hansen, 1958):

$$sin\nu = \frac{-(\dot{\varepsilon}_1 + \dot{\varepsilon}_3)}{\dot{\varepsilon}_1 - \dot{\varepsilon}_3}$$

or

$$sin\nu = \frac{\dot{\varepsilon}_v}{\dot{\varepsilon}_v - 2\dot{\varepsilon}_1} \qquad (2)$$

Formula (2) can be used in the plane state and in the triaxial state as well. A close connection exists between φ', c' and ν. This will be demonstrated for a pure frictional material such as silt, sand or gravel.

Basic properties of sand minerals

The most simple way of studying the basic physical properties of a mineral during shear is to use two blocks, with plane surfaces, and force them to slide against each other. By changing the normal stress σ' from test to test and measuring the shear stress τ, it can be observed that Coulomb's failure criterion is valid:

$$\tau \leq c_\mu + \sigma tan\varphi_\mu \tag{3}$$

where c_μ is the mineral cohesion and φ_μ is the mineral friction. The mineral's cohesion and friction depend on the chemistry and roughness of the surface and on the fluid into which it is submerged. Sand normally consists of grains of quartz mineral, but may also contain grains of feldspar. Sand minerals are pure frictional materials: $c_\mu = 0$.

Rowe (1962) reported shear box tests where a free mass of particles were forced to slide over a plane surface of the same mineral. During some tests the particles were fixed, in order to demonstrate the absence of any influence of rolling. In tests where quartz particles were fully saturated by water, values of φ_μ between 22° and 30° were observed.

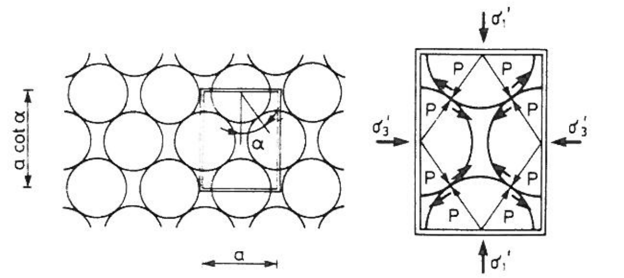

Fig. 4. Idealized assembly of sand grains in plane strain state. A regular packing of cylinders and a basic element.

Basic properties of particle assemblies

Rowe (1962) proposed the simplest way of studying the basic properties of an assembly of particles. In the plane stress state, his model consists of a regular packing of cohesionless uniform cylinders (Fig. 4). The geometry of the packing can be completely described by the angle α and the basic elements can be defined as having four hemispheres with four contact points.

The normal stresses on the surface of a basic element produces contact forces between the cylinders. During compression the cylinders will slide against each other. The normal force in the contact points is P and the shear force is then $P \cdot tan\varphi_\mu$.

If the length of a horizontal side is 1, then the length of the vertical side is $cot\alpha$.

Since horizontal and vertical equilibrium are required:

$$\sigma_3 cot\alpha = 2P sin\alpha - 2P tan\varphi_\mu cos\alpha$$

$$\sigma_1 = 2P cos\alpha + 2P tan\varphi_\mu sin\alpha$$

which can be combined in

$$\frac{\sigma_1}{\sigma_3} = cot\alpha cot(\alpha - \varphi_\mu) \tag{4}$$

which has been demonstrated by Rowe. The angle of dilation ν (formula (2)) can now be introduced in formula (4) in a very simple way.

During sliding no elongations take place in directions inclining α with the vertical, corresponding to point C and D at the Mohr's circle for strain increments (Fig. 3 and 4). It is seen that

$$\nu + 2\alpha = 90°$$

and then

$$\frac{\sigma_1}{\sigma_3} = tan(45 + \frac{\nu}{2})tan(45 + \frac{\nu}{2} + \varphi_\mu) = tan^2(45 + \frac{\varphi'_s}{2}) \tag{5}$$

The effective friction angle φ'_s of the assembly of particles is introduced in formula (5). φ'_s is defined by

$$sin\varphi'_s = (\frac{\sigma'_1 - \sigma'_3}{\sigma'_1 + \sigma'_3})_{failure}$$

Formula (5) then shows that φ'_s depends on the material friction angle φ_μ and the dilation angle ν.

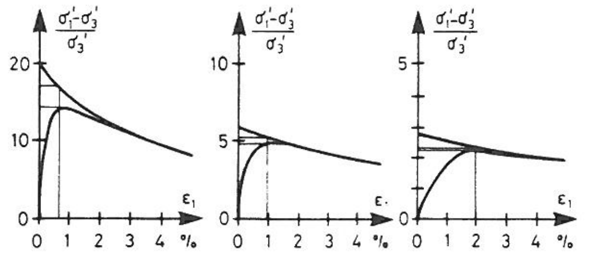

Fig. 5. Idealized relationship between strength, stresses and vertical strain. a) dense packing. b) medium packing. c) loose packing.

The vertical strain ε_1 depends on increment of the angle α. Therefore, the cylinder model can be used to calculate stress - strain curves for an assembly of cylinders with perfect plastic behaviour. This is demonstrated in Fig. 5. The actual test curves should then show a rather sharp peak point densely packed and a rather smooth curve when loosely packed.

The regular packing model does not accurately reflect reality, therefore strongly limiting the use of formula (5). Regular packing restricts the principal stresses to two directions - in Fig. 4 vertical and horizontal - and distinct failure surfaces can also develop in two directions only. If the packing is irregular, containing particles of different sizes and shapes, the particles may move freely.

Triaxial tests

A distinct failure surface can be prevented during triaxial testing by using specimens with heights equal to their diameters; this homogeniously distributes the strains over the entire sample. Therefore, formulas based on a regular packing might be used to describe the connection between φ_μ, ν and φ'_s.

Formula 5 is based on a regular packing of cylinders but Rowe (1962) has shown that spheres in regular packings lead to the same formula.

Analysed in Fig. 6 - 8 are test results from two series of triaxial tests which were performed on a fairly uniform quartz sand (Lund No. 0).

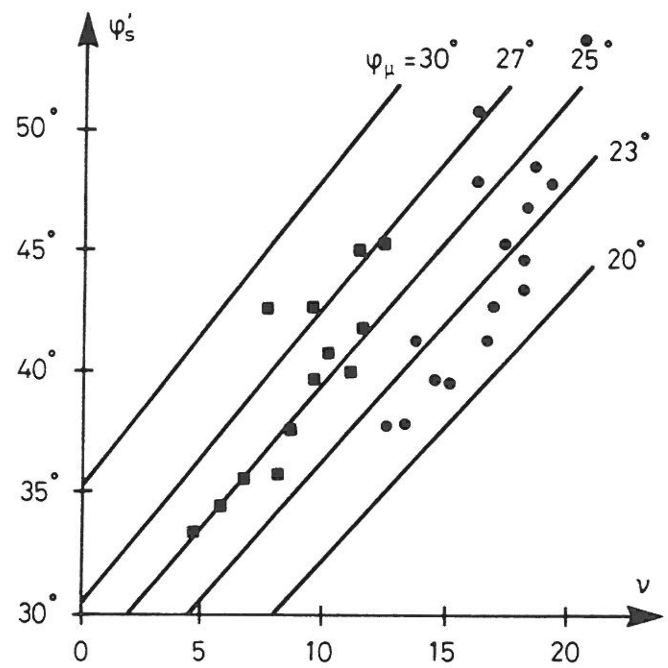

Fig. 6. Triaxial tests on Lund No. 1 sand. φ'_s as function of φ_μ and ν.

In agreement with results reported by Rowe (1971) (and mentioned earlier in this paper) φ_μ, as calculated in formula (5) varies between 21° and 29°. Fig. 7 shows that φ_μ depends on the void ratio. A possible explanation is that φ'_s is based on the maximum stress ratio. As the void ratio is decreased, the gab between the real curve at its maximum load and the idealized curve is increased. This is illustrated in Fig. 5. From this it may be concluded that the test results are in close agreement with formula 5. Fig. 8 shows that ν decreases as the stress level increases. It probably vanishes at a charateristic high stress level, which is a function of the void ratio.

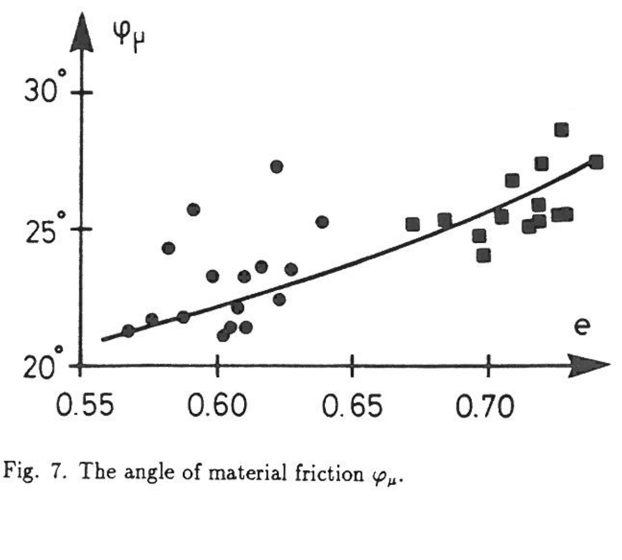

Fig. 7. The angle of material friction φ_μ.

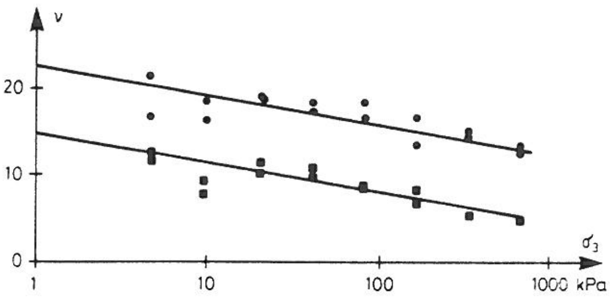

Fig. 8. The angle of dilation ν versus minor principal stress σ'_3.

Assuming that the mineral friction φ_μ is a constant and the mineral cohesion $c_\mu = 0$, then the following three points can be made when comparing Fig. 8 and formula 5:

i) if ν is a constant then φ'_s must also be a constant and the soil mass then behaves as a pure frictional material.

ii) if $\nu \rightarrow 90°$ for $\sigma_3 \rightarrow 0$ and decreases with increasing stress level first rapidly to around 30° and then more slowly to 22° at very high stress levels then the sand mass may behave as a cohesional and frictional material.

iii) if ν decreases slowly with increasing stress levels the failure line would be a curve which is downwards concave. This last possibility agrees well with Fig. 8. An idealized curved failure line is calculated in Fig. 9 using formula (5) and Coulomb's failure criterion. The parameters are taken from Fig. 7 and 8 for a dense sand: $\varphi_\mu = 23°$ and $\nu = 23 - 3.5log\sigma'_3$.

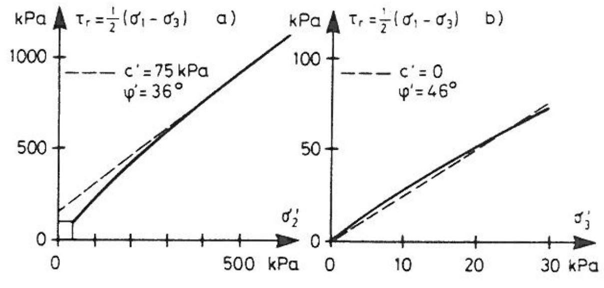

Fig. 9. $\tau - \sigma$ diagram calculated from formula (5) and Fig. 8.

Analytical and numerical analysis

The curved failure line is most simply described by the tangent angle of friction φ'_t and the tangent cohesion c'_t, which corresponds to a characteristic stress level σ'_c.

c'_t is often called "apparant" since the mineral itself has no cohesion ($c_\mu = 0$) and because it vanishes at small stress levels. c'_t is caused by the amount of packing and not on the type of mineral.

In theoretical or numerical analysis the "apparant" cohesion c'_t is often neglected, but it should be used in order to cover the failure curve in the best possible way. It is also important to take the stress level into consideration.

Meyerhof (1948) used a characteristic stress level of 10 per cent of the ultimate failure load calculating small scale tests. Perhaps an improved method would be to use a stress level σ_c which leads to the minimum bearing capacity.

The stress level influence can also be taken into account by modifying the Coulomb's failure criterion from

$$\tau_r = \frac{1}{2}(\sigma_1' - \sigma_3') = \frac{sin\varphi'}{1 - sin\varphi'}(\sigma_3' + c'cot\varphi') \quad (6)$$

to

$$\tau_r = \frac{1}{2}(\sigma_1' - \sigma_3') = \frac{sin\varphi_a}{1 - sin\varphi_a}\sigma_3'(1 + \frac{c_a cot\varphi_a}{m \cdot \sigma_3'})^m \quad (7)$$

The formula (6) is chosen because the minor principal stress $\sigma_3' \geq 0$. The suffix a indicates asymptotic parameters. m describes the curvature of the failure lines. For a perfect frictional sand mass $m = 0$. For a perfect cohesional and frictional sand mass $m = 1$ and a curved failure line is defined by $0 \leq m \leq 1$. (Jacobsen, 1970).

Fig. 9 shows the failure curve based on formula (5) using $\varphi_\mu = 23°$ and $\nu = 23° - 3.5 log\sigma_3$ as mentioned earlier. But the same curve can also be obtained by using formula (7) with $\varphi_a' = 37\overset{\circ}{.}5, c_a = 68kPa$ and $m = 0.14$. The agreement between the two formulas is nearly perfect.

Direct use of formula (7) is rather complicated. But the friction angle φ_t' and the cohesion c_t' can be estimated as functions of the stress level (or of the minor principal stress σ_3') by differentiation of formula (7).

Model testing

The stress level in small scale model tests is often much lower than that of the prototype. When the stress level is low, the failure curve bends away from the straight failure line, thus the model has results with higher φ_t' values and lower c_t' values than in the prototype. Fig. 9 shows the failure line and strength parameters for the prototype and in the model respectively. Therefore, small scale model tests performed on the surface of an unloaded sand mass will considerably overestimate the bearing capacity of a foundation.

When describing model tests in scientific papers, the characteristic stress level corresponding to φ_t' and c_t' should be noted in addition to the grain size distribution curve, the void ratio and the results from triaxial testing.

In most cases only an angle of effective friction φ_s' is stated making it very difficult to compare results from different test series.

The parameter m

Test results show parameter m to remain constant for a specific type of sand while φ_a' and c_a' are depending on the void ratio. Fig. 10 illustrates the combined results of the following test series: Four series on fine to medium graded sand ($U \sim 2$), one test series on sandy gravel ($U \sim 10$) and one test series on silty sandy glacial till ($U \sim 100$). It is obvious from this that m depends on the uniformity coefficient U.

The previous use of $m = 0.14$ (in formula (7)) corresponding to an assembly of uniform spheres or cylinders ($U = 1$) is in close agreement with the results shown in Fig. 10.

Therefore, the curved failure line can be estimated at a corresponding stress level from the grain size distribution curve and the angle of friction φ_s'.

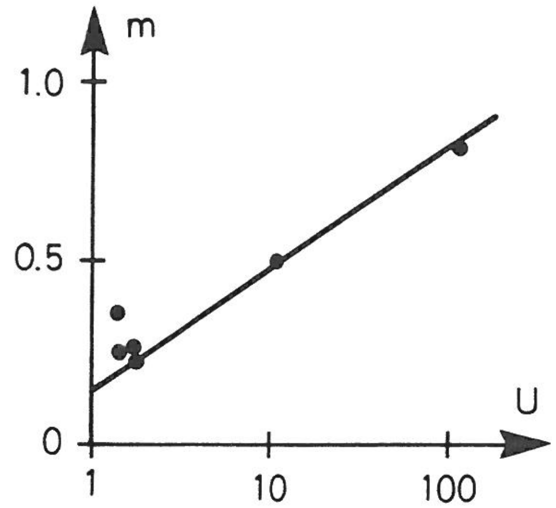

Fig. 10. The parameter m versus coefficient of uniformity U.

Conclusions

a) The soil behaviour during failure is shown to depend on the stress state. A failure criterion describing this was proposed by Lade (1972). However, for theoretical analysis in plane stress states, Coulomb's failure criterion can be used with sufficient accuracy if a plane angle of friction φ_{pl} is used in place of the triaxial angle of friction φ_{tr}. φ_{pl} is about 10 per cent higher than φ_{tr} (formula (1)).

b) The behaviour of a regular packing of cylinders or spheres has been studied by Rowe, 1962. By introducing the angle of dilation ν into this model, a very simple formula (5) is derived which connects the secant angle of friction φ_s' and ν to the angle of friction φ_μ of the mineral itself. Triaxial tests with a uniform sand show close agreement with this theory.

c) It has been shown that the cohesion c' is due to the packing of sand grains, and not due to the type of mineral. In medium to dense packings, the dilatancy of the sand during failure causes the failure line to be curved. In order to best describe this curve when using Coulomb's failure criterion, the tangent to the curve at any given stress level, should be used. Thus, the tangent angle of friction φ_t' and the tangent cohesion c_t' depend on the stress level. In theoretical analysis these values should be used.

d) The curved failure line can be described by an extended Coulomb's failure criterion (formula (7)), by introducing a parameter m to describe the curvature. Test series demonstrate that this parameter m depends on the uniformity coefficient U. When the test results (Fig. 10) are extrapolated to $U = 1$, a value of m is determined. This is found to correspond with the value of m calculated from the regular packing model.

e) Formula (7) does not play a significant role in engineering practice, but it does point out the importance of performing laboratory tests and small scale model tests at the same stress levels as those of the actual foundations.

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